Online Semester End Exam December 2020
Semester –III-S.Y.BSc
Paper 1- Calculus III(USMT301)

1.	If $u(x,y,z) = \log(x^2y - xy^2 + y^2z - yz^2 + z^2x - zx^2)$. Then $u_x + u_y + u_z$ is $\left(Here\ u_x = \frac{\partial u}{\partial x}\right)$ (a) 1 (b) 0 (c) -1 (d) 2
Ans	b) 0
2.	If $u(x,y) = y^n f(x/y)$ then $x^2u_{xx} + 2xyu_{xy} + y^2u_{yy}$ is $\left(Here\ u_{xy} = \frac{\partial^2 y}{\partial y \partial x}\right)$ (a) $n(n+1)u$ (b) $n(n-1)u$ (c) $n(n-2)u$ (d) 0
Ans	(b) $n(n-1)u$
3.	If $u(x,y,z) = \log(x^2 + y^2 + z^2 - 3xyz)$. Then $x^2u_{xx} + y^2u_{yy} + z^2u_{zz} + 2xyu_{xy} + 2yzu_{yz} + 2zxu_{zz}$ is (a) -3 (b) 3 (c) -1 (d) 0
Ans	(a)0
4.	$f : [0,1] \to \mathbb{R}^2, f(t) = (t^2, t^3)$. Then the value of θ satisfying mean value property is (a) 0 (b) $1/2$ (c) $1/3$ (d) does not exist
Ans	(d) does not exist
5.	$f: \mathbb{R}^3 \to \mathbb{R}$ is a differentiable function such that $\frac{\partial f}{\partial y} = 0$, then (a) $f(x,y,z) = f(x,y',z), \forall\ (x,y,z),(x,y',z) \in \mathbb{R}^3$. (b) f is independent of x and z. (c) f is constant (d)

	None of the above
Ans	b) f is independent of x and z.
6.	The linear approximation to $\frac{1}{1-x-2y}$ near the origin is (a) $1+x+2y$ (b) $1+x-2y$ (c) $1-x+2y$ (d) None of these
Ans	(a) 1+x+2y
7.	(r,θ) are the polar coordinates of a point $P(x,y)$ in the plane then (a) $\frac{\partial x}{\partial r}=\frac{\partial x}{\partial \theta}$ (b) $\frac{\partial x}{\partial r}=\frac{x}{r}$ (c) $\frac{\partial x}{\partial r}=\frac{1}{\frac{\partial r}{\partial x}}$ (d) None of these
Ans	(b) $\frac{\partial x}{\partial r}=\frac{x}{r}$
8.	$f(x,y)=0$. Then $\frac{dy}{dx}$ equals (a) $\frac{\partial f}{\partial x}/\frac{\partial f}{\partial y}$ (b) $\frac{\partial f}{\partial y}/\frac{\partial f}{\partial x}$ (c) $-\frac{\partial f}{\partial x}/\frac{\partial f}{\partial y}$ (d) None of these
Ans	(c) $-\frac{\partial f}{\partial x}/\frac{\partial f}{\partial y}$
9.	$f:\mathbb{R}^2\to\mathbb{R}^2$ given by $f(x,y)=(x+a,y)$. Four statement are given below i. f is differentiable on $\mathbb{R}$ except at $(0,0)$ ii. The total derivative of f at any point $(a,b)\in\mathbb{R}^2$ exists iii. The total derivative of f at any point $(a,b)\in\mathbb{R}^2$ is the identity matrix of order 2. iv. The total derivative of f at any point $(a,b)\in\mathbb{R}^2$ is given by the matrix $\begin{bmatrix}1+1 & 0\\ 0 & 1\end{bmatrix}$. Choose the correct alternative. (a) Only (i) is true (b) (ii) and (iii) are true (c) (ii) and (iv) are true (d) Only (ii) is true

Ans	(b) (ii) and (iii) are true
10.	$f: \mathbb{R}^2 \to \mathbb{R}\ f(x,y) = x + y, g: \mathbb{R}^2 \to \mathbb{R}; g(x,y) = k, k$ is constant. Then the total derivative of the map fg at a point (a,b) denote by $D(f\ g)(a\ b)$ (a) $k(a+b) + (a+b)$ (b) 0 (c) $k(a+b)$ (d) does not exist
Ans	(c) $k(a+b)$
11.	$f: \mathbb{R}^3 \to \mathbb{R}$ is such $f(x,y,z)$ is a non zero constant for all points on a path $\propto (t)$. Then (a) the vectors $\nabla f\left(\alpha(t)\right)$ and $\alpha'(t)$ are parallel (b) the vectors $\nabla f\left(\alpha(t)\right)$ and $\alpha'(t)$ are perpendicular. (c) $\nabla f(\alpha(t)) = k a'(t)$ (d) $\nabla f\ (\alpha(t)) = a'(t) = k, k \neq 0$
Ans	(d) $\nabla f\ (\alpha(t)) = a'(t) = k, k \neq 0$
12.	$g: [0,1] \to \mathbb{R}^n; g(t) = u + tw$ where u, w are fixed vectors in R^n $f: \mathbb{R}^n \to \mathbb{R}^m$ is a differentiable function. If $F(t) = f(g(t))$ then total derivative of F at a point t, $DF(t)$, is the (a) $Df(t)(u+tw)$ (b) $Df(u+tw)(w)$ (c) $Df(t)(\omega)$ (d) $Df(u+tw)(u+tw)$
Ans	d) $Df(u+tw)(u+tw)$
13	The distance between u=(2,-2, 1) and v=(-1,0,0) is (a)$\sqrt{15}$ (b)15 (c) $\sqrt{14}$ (d)14
14	Considering x.y as inner product in $\mathbb{R}^n$, complete the statement $\|\alpha\|\sqrt{x.x} =$ (a) $\|\alpha x\|$

	(b) $\sqrt{\alpha}\|x\|$ (c) $\alpha.x$ (d) $\|\alpha x^2\|$
15	The norm of the vector u= (-4,-2,0) is (a) 1 (b) $\sqrt{20}$ (c) 0 (d) $\sqrt{2}$
16	Find $\lim_{(x,y)\to(4,\pi)} x^2 \sin\frac{y}{8}$ a) $8\sqrt{2}$ b) 4/ π c) $8/\sqrt{2}$ d) 0
17	If a= (-1.1), v=(3,4) and $f(x,y) = 3x - 2y$ then f(a+v) = (a) 4 (b)-4 (c)3 (d)5
18	Compute the partial derivative of the function $f(x,y) = e^{xy^2} + yx^3$ with respect to x at the point (-1, 0). (a) -1 (b) −1/e (c) 0 (d) -1
19	Find the domain and range of this function z= x sin 1/y

	(a) D: x > 0, y > 0, R: z ≥ 0 (b) D: All Reals , R: z ≥1 (c) D: All Reals , R: all reals (d) D: x > 0, y > 0, R: z ≥1
20	The limit of the sequence $a_n = (\frac{2n+1}{n}, \frac{1}{n^2})$ is (a) (0.0) (b) (2,0) (c) (2,1) (d)(2,1)
21	Which of the following statements is true? (a) Every convergent sequence in $\mathbb{R}^n$ has a unique limit. (b) Every bounded sequence in $\mathbb{R}^n$ converges to its boundary point . (c) Every convergent sequence in $\mathbb{R}^n$ may converge to more than one point . (d) Every bounded sequence in $\mathbb{R}^n$ is has unique limit
22	Let $f\colon \mathbb{R}^2 \to \mathbb{R}$ be defined as $f(x,y) = \begin{cases} \frac{xy^3}{x^2+y^6}, if\ (x,y) \neq (0,0) \\ 0, if\ (x,y) = (0,0) \end{cases}$, $and\ l = \lim_{(x,y)\to(0,0)} f(x,y)$. Then (a) l does not exist (b) l exists along the path $y = mx^n$ only if $n > 3$ (c) f has removable discontinuity at (0,0) (d) None of these
23	If iterated limits exist and $\lim_{x\to a}\lim_{y\to b} f(x,y) = \lim_{y\to b}\lim_{x\to a} f(x,y)\ then$ a) f is continuous

	(b) f may or may not be continuous (c) f is discontinuous (d) nothing can be said
24	The directional derivative of $f(x, y) = x^2 + 2xy$ at (0, 1) in the direction towards (1, −1) is (a) 0 (b) $1/\sqrt{2}$ (c) $-2/\sqrt{2}$ (d) $\sqrt{2}$
25	$u(x, y) = x^2 + y^2$, $x = r + e^s$, $y = \log s$ then $\partial u / \partial r$ a) $r + e^s$ b) $2r + 2e^s$ c) r e^s
Ans	a) $2r + 2e^s$
26	f : R 2 → R 2 given by f(x, y) = (x , y+1). Four statement are given below i) f is differentiable on R except at (0,0) ii) The total derivative of f at any point (a, b) ∈ R^2 exists iii) The total derivative of f at any point (a, b) ∈ R^2 is the identity matrix of order iv) The total derivative of f at any point (a, b) ∈ R^2 is given by the matrix $\begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$ Choose the correct alternative. (a) Only (i) is true. (b) (ii) and (iii) are true. (c) (ii) and (iv) are true. (d) Only (ii) is true.
	(b) (ii) and (iii) are true.
27	If A = fxx(a, b), B = fxy(a, b), C = fyy(a, b), then f(x, y) will have a minimum at (a, b) if (a) fx = 0, fy = 0, $AC < B^2$ and $A < 0$. (b) fx = 0, fy = 0, $AC = B^2$ and $A > 0$.

	(c) fx = 0, fy = 0, $AC > B^2$ and $A > 0$. (d) fx = 0, fy = 0, $AC > B^2$ and $A < 0$.
Ans	(c) fx = 0, fy = 0, $AC > B^2$ and $A > 0$.
28	The linear approximation to $(1 - x - 2y)$ near the origin is a) 1-x -2y b) 1+x+2y c) 1+x-2y d) 1-x+2y
29	If f: $R^m \to R^n$ and f=($f_{1,}$ f2,f3,…..f_n) fi is differentiable then (a) Each f is differentiable (b) Some f is differentiable (c) Some f is discontinuous Each f is not continuous
Ans	Each f is differentiable
30	If $(f_{xx})(f_{yy}) - (fxy)^2 < 0$, then Function has a) no extreme value b) extreme value c) may extreme value d) none of these
Ans	no extreme value
31	Given f (x, y) =e^x siny, what is the value of the second term in Taylor's series near (1,0) where it is expanded in increasing order of degree a) x-1 b) (y-1)e c) (x-1)e/2 d) ye
Ans	d) ye
32	Consider the f(x, y) = $x^2 + y^2 - 8$. For what values of a do we have critical points for the function. a) independent of 8

	b) for any real number except zero c) depend on 8 d) 8
	(a) independent of 8
33	The point (0,0) in the domain of f(x, y) = cos(xy) is a point of ___________ a) Saddle b) Minima c) Maxima d) Constant
Ans	a) Saddle
34	**Hessian matrix of the function** *f*(*x*,*y*,*z*)=*x3*+*y3* a) $\begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix}$ b) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ c) $\begin{bmatrix} 0 & 6 \\ 6 & 0 \end{bmatrix}$ $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$
Ans	$\begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix}$
35	Critical point of f(x, y) = x^3+3x+y^3+3y is a) (3,3) b) (1 ,1) c) (½, ½) d) (2,1/2)
Ans	b) (1 ,1)